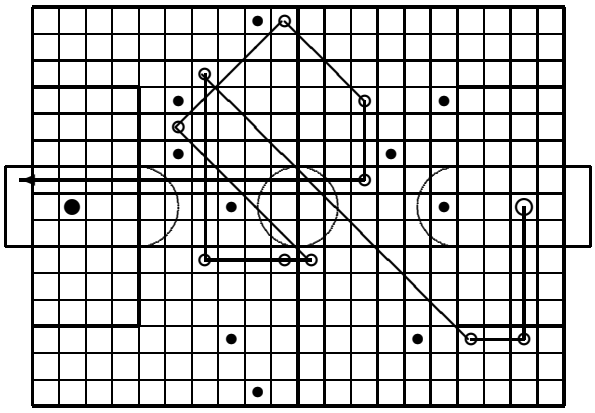
Upper Elementary School Mathematics

A textbook based on the International Mathematics Competition : Upper Elementary School Division 2003 – 2014



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This book is primarily written as an ambitious textbook for upper elementary school mathematics. Problems posed in the Upper Elementary School Division of the International Mathematics Competition from 2003 to 2014 are usd as illustrative examples. There is a full description of the I. M. C., and the problems are listed year by year, with a cross-reference index. This may also serve as a classroom resource, and is well-suited for self-study.

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Introduction

This volume serves two purposes. On the one hand, it is a compilation of the problems in the Upper Elementary School Division of the International Mathematics Competition, from its inception in 2003 to 2014. On the other hand, it is written as a mathematics textbook, albeit with plenty of extra-curricular material, for the upper elementary school.

The prerequisite for this book is a good grounding in lower elementary school mathematics. We presuppose that the student readers are familiar with the counting numbers 1, 2, 3, ..., called positive integers here, along with the number 0. They should be reasonably competent in the four basic operations of addition, subtraction, multiplication and division. Finally, they should be comfortable with digits and placement values in the base-ten system.

Some understanding of simple upper elementary school mathematics is also desirable. The students readers should know the terms divisors and multiples. They should have been exposed to both ways of handling inexact divisions. The first way is via long divisions, with quotients and remainders. The second way involves the introduction of fractions, along with the terms like numerators and denominators. They should have some ideas about decimals, ratios and percentages. They should know about powers, in particular squares and cubes, as well as integral square and cube roots. Finally, they should have general notions of geometric terms such as points, lines, triangles and polygons.

The main part of the book deals with arithmetic, and we have avoided both the language and the methods of algebra. There is substantial material on geometry. Additional topics include the methods of counting, as well as recreational mathematics. We give without proof a number of more advanced results such as the Fundamental Theorem of Arithmetic about the existence and uniqueness of prime factorizations.

For those primarily interested in the competition itself, the contest papers are given in Appendix A. All questions, and only these ones, are used as examples within the text. In some of the questions, the wording have been changed from the actual competition. To locate solutions, use the Problem Index for cross reference.

Part of the immense mathematical legacy of the great **Martin Gardner** is the subject of Appendix B.

There is a shortage of material in mathematics at the upper elementary school level. This book is an attempt to somewhat fill this void.

Andy Liu,
Canada,
December, 2014.

International Mathematics Competition

1. History

This year, 2014, marks the twelfth anniversary of our competition. During this period of over a decade, our competition has enjoyed tremendous growth.

Pramote Kajornpai of Thailand, was the founder of the International Mathematics Competition, abbreviated to IMC, in 2008. He merged two existing competitions when he brought both of them to Chiang Mai, a famous mountain resort in Thailand. The predecessor of the Junior High School Division was the Invitational World Youth Mathematics Inter-city Competition, which is covered in another book. The predecessor of the Upper Elementary School Division was the Elementary Mathematics International Contest.

This was founded in 2003 in Nakhon Pathom, Thailand, also by Pramote. It came about because the Invitational World Youth Mathematics Inter-city Competition was cancelled that year because of the SARS scare. It was decided that a new competition should be started in its place. However, in order not to compete against each other in future years, the new contest is targeted at upper elementary school students. Participating countries are Brunei, Bulgaria, China, Indonesia, Iran, Lao, Malaysia, the Philippines, Singapore, Taiwan, Thailand, the United States and Vietnam.

The 2004 event was hosted by Lucknow, India, joining the competition along with Pakistan and South Africa, in place of Brunei, China, Lao, Malaysia, Singapore and the United States. In 2005, the competition went to Cebu, in the Visayas Region of the Philippines. Iran, Pakistan and Vietnam were absent. New entries are from Cyprus and Hong Kong. China and Malaysia also returned.

In 2006, the host city was Denpasar, Indonesia, in the famous resort island of Bali. China and Malaysia did not return, but Sri Lanka joined. In 2007, the competition went to Hong Kong, China. The cast was exactly the same as in 2005, with China and Malaysia replacing Sri Lanka.

In the landmark year of 2008, the number of participating countries shot all the way up to eighteen, with the return of Iran. New countries are Bangladesh, Brunei, Laos, Nigeria, Rwanda and South Korea. In a trial experiment, the two Divisions of the IMC were held in separate locations in 2009. The host city for the Upper Elementary School Division was Iloilo City, also in the Visayas Region of the Philippines. Bangladesh, India, Iran, Laos and Rwanda were absent, but Mongolia and Nepal joined. The experiment was abandoned after one year.

Incheon, South Korea, hosted the event in 2010. The number of participating countries moved back up to seventeen, with the return of India and Iran. In 2011, the host city was Denpasar, Indonesia, for the second time. A new attendance record of nineteen was set, and this trend will continue in the next three years. Nepal and Nigeria were absent, but Sri Lanka, the United States and Vietnam returned while Russia joined for the first time. In 2012, the event went to Taipei, Taiwan. Brunei was absent but Nigeria returned. Newcomers were Australia, Singapore and the Ukraine.

In 2013, the competition moved to Europe for the first time. The host city was Burgas, Bulgaria, a beautiful Black Sea resort. Australia, Singapore and the Ukraine were absent but Nepal returned. Newcomers were Canada, Kazakhstan, Macau and Romania. In 2014, the host city was Daejeon, South Korea. Despite the absence of Romania, Russia and the United States, the number of participating countries rose to twenty-five, with newcomers Mexico and Uzbekistan joining returnees Brunei and Singapore.

2. Format

The Competition consists of an Individual Contest and a Team Contest. The problems are submitted by participating countries, and selected by a Central Academic Committee in conjunction with a Local Academic Committee. The official languages are English, Chinese and the language of the host, if different from English and Chinese. However, students may write in their own native languages.

The problems are the heart and soul of any mathematics competition. Over the years, the participating countries have contributed many beautiful problems. Great care is exercised in problem selection, and much further work is done in polishing those which have been chosen. Thus the archive of this competition is a most valuable resource.

The Individual Contest is a fairly tradition competition, and its format has remained practically the same. The focus is on testing the participants' basic mathematics skill. Time allowed is 90 minutes. The paper consists of 15 questions in which only answers are to be given. Each question is worth 10 points for a total of 150 points. No part marks are given, and there is no penalty for wrong answers. No calculating or electronic devices are allowed.

The following Individual Awards are given based on performance in the Individual Contest. The fraction of students receiving Gold Medals is $\frac{1}{15}$. The fraction of students receiving Silver Medals $\frac{2}{15}$. The fraction of students receiving Bronze Medals is $\frac{3}{15} = \frac{1}{5}$. The fraction of students receiving a Certificate of Merit is $\frac{4}{15}$. Overall, $\frac{6}{15} = \frac{2}{5}$ of the students receive medals, and $\frac{10}{15} = \frac{2}{3}$ receive awards. Of course, all fractions have to be rounded off, according to the total number of students.

The Team Contest is an innovation, and its format, time allowed and number of problems varied. Standardization has been achieved in recent years. Here, our aim is to promote thinking beyond the curriculum and impress upon the students that mathematics is much more than a set of well-versed algorithms. Most of the problems have unorthodox settings and elegant ideas behind them. Many also have some element of play in them, which is essential to the full enjoyment of the subject.

The paper consists of 10 problems each worth 40 points, for a total of 400. Only answers are required for odd-numbered problems, but full solutions are required for even-numbered problems. Part marks may be awarded. As in the Individual Contest, no calculating or electronic devices are allowed.

The total time allowed is only 60 minutes. During the first 10 minutes, the four team members discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. No writing is allowed at this point. In the next 35 minutes, the team members go their separate ways to write the solutions of their allotted problems, with no further communications among them. During the last 15 minutes, the four team members reconvene and try to solve the last 2 problems together.

There are two Team Awards, one based on the performance in the Team Contest, and the other based on the performance in the Individual Contest. For both awards, the teams are randomly drawn into six groups. The teams within each group compete for one Gold Trophy, two Silver Trophies and three Bronze Trophies. For the award based on the performance in the Individual Contest, the team score is the sum of the best three individual scores among the four team members.

There are also a Grand Champion Trophy, a First Runner-up Trophy and a Second Runner-up Trophy. Here, the team score is the sum of the scores in the Individual Contest of all four team members, plus the team's score in the Team Contest.

3. Structure

A Central Executive Organization is responsible to soliciting host countries. There are no registration fees, and local expenses such as room and board, transportation and official excursions are free. Funding is raised by a Local Executive Organization. With more and more countries joining the competition, each is now limited to sending two teams of four students. The host country may enter ten teams. The country which will be hosting the next competition, as well as all former host countries, are allowed four teams. To increase further participation from within, the host country may hold a parallel competition for local students.

The event typically lasts five days. Monday is the day of arrival. Leaders of teams whose students will not be writing the competition in any of the official languages will be engaged in the translation process, with members of the Central Academic Committee on hand for consultation. This may continue into Tuesday when the Opening Ceremony is held. On that day, there is also a public lecture for students and a seminar for leaders and deputy leaders.

Wednesday is the Competition Day, with the Individual Contest in the morning and the Team Contest in the afternoon. The afternoon continues with a fun session of mechanical puzzles provided by the Chiu Chang Mathematics Foundation of Taiwan, featuring mainly products from the US company ThinkFun. Later comes a most important feature of the event, the Cultural Evening during which teams give a short performance based on their national heritage. In recent years, the Cultural Evening spills over to Thursday evening as well.

Thursday is the Excursion Day, during which the Local Academic Committee continues with the grading which has started the day before. Graded papers are returned to the team leaders upon their return. Any dispute is adjudicated by the Central Academic Committee. If it is still unresolved, a Jury meeting of all team leaders is convened to make a final decision.

The only item in the official itinerary on Friday is the Closing Ceremony in the morning. Apart from academic awards, there is a trophy for the team judged to have given the best performance in the Cultural Evening, and three or four other trophies based on social events. A certificate of participation is given to each team leader and each team member. Departure usually commences in the afternoon, but some host allows teams to stay on until Saturday.